Ambiguous Case for Law of Sines

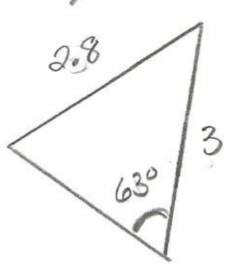
* Law of Sines: needs ASA or AAS

* Law of Cosines: needs SAS or SSS

* What happens when given "SSA"
  - given two sides & the angle at the end of one of the sides

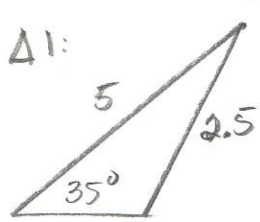
* Given "SSA", three cases arise
  1. One unique triangle
  2. Two triangles
  3. No triangle

<u>Case 1:</u> there can be one unique triange

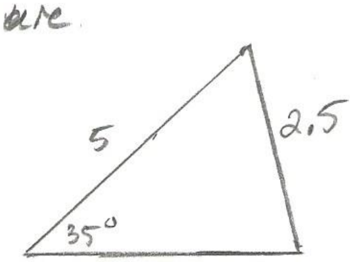


<u>Case 2:</u> two triangles with same side lengths and angle measure

Δ1:

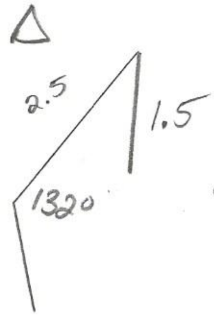


Δ2:



- Both Δ's have same "SSA" measures but two different Δ's

<u>Case 3:</u> No Δ



- Could be given measurements that don't even make a Δ.

Ambiguous Case

given "SSA" or $\angle A = 40^\circ$ $a = 6$ $b = 9$

1st: want to determine how many Δ's

I. if $\angle Z < 90^\circ$

Given $\angle Z$, sides Z & y

- $z < y$
  - $z < y \cdot \sin Z$ → no sol (No Δ)
  - $z = y \cdot \sin Z$ → one sol (1 Δ)
  - $z > y \cdot \sin Z$ → 2 sol (2 Δ)
- $z \geq y$ → one solution (1 Δ)

II. if $\angle Z \geq 90^\circ$

- $z \leq y$ → No sol (No Δ)
- $z > y$ → One Sol (1 Δ)

* Given the measurements follow the chart accordingly

* the chart is relative depending on what angle given

Ambiguous Case Examples

Example given $A = 42°$, $a = 12$, $b = 22$

- since $A < 90°$, use top of chart
- $a < b$, use left side
- now compare a __ $b \cdot \sin A$

$$12 \ \_\_ \ 22 \cdot \sin 42°$$

$$12 < 14.2$$

* since left side is less, then no solution, there is no Δ

Example given $C = 50°$, $c = 8$, $a = 5$

- since given $\angle C$, side c is the left letter

- since $C < 90°$, use top part
- since $c > a$, use right side and there is one sol. (one Δ)

Finding the Δ:

* use Law of sines

$A = 29°$ $B = 101°$ $C = 50°$

$\frac{\sin 50}{8} = \frac{\sin A}{5}$ $\angle A = 29°$

$a = 5$ $b = 10.3$ $c = 8$

$\frac{\sin 101}{b} = \frac{\sin 50}{8}$ $b = 10.3$

Ambigious Case

Example: given $B = 50°$, $b = 13$, $a = 15$

- $B < 90$ top of chart
- $b < a$, left side
- $13 > 15 \cdot \sin 50$, $\therefore$ 2 Δ's

Δ I:

$A = 62°$ $B = 50°$ $C = 68°$

$a = 15$ $b = 13$ $c = 15.7$

- $\frac{\sin A}{15} = \frac{\sin 50}{13}$

  $\sin A = \frac{15 \sin 50}{13}$

  $A = 62°$

- $\frac{\sin 68}{c} = \frac{\sin 50}{13}$

  $c = \frac{13 \sin 68}{\sin 50}$

  $c = 15.7$

Δ II:

$A = 118°$ $B = 50°$ $C = 9°$

$a = 15$ $b = 13$ $c = 9$

- Given still same
- from 1st Δ, take $\angle A$ since that was found first and subtract from 180.

$\angle A = 180 - 62$

$\angle A = 118$

- $\angle C = 180 - 118 - 50$

  $\angle C = 9°$

- $\frac{\sin 50}{13} = \frac{\sin 9}{c}$

  $c = 9$