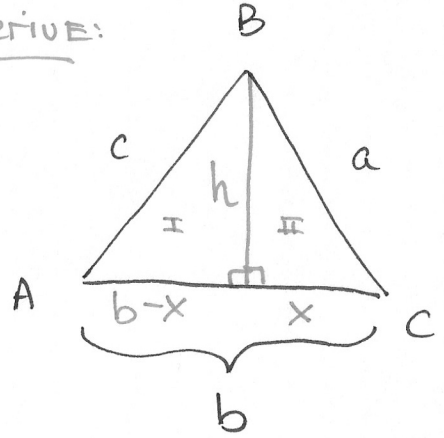
LAW OF SINES: AAS / ASA

LAW OF COSINES: SAS / SSS

at least 2 sides

Derive:



$\cos C = \frac{x}{a}$

$x = a\cos C$

△I: $h^2 + (b-x)^2 = c^2$

$h^2 = c^2 - (b-x)^2$

△II: $x^2 + h^2 = a^2$

$h^2 = a^2 - x^2$

same

$c^2 - (b-x)^2 = a^2 - x^2$

$c^2 - (b^2 - 2bx + x^2) = a^2 - x^2$

$c^2 - b^2 + 2bx - x^2 = a^2 - x^2$

$c^2 - b^2 + 2bx = a^2$

$c^2 - b^2 + 2ba\cos C = a^2$

$* \; c^2 = a^2 + b^2 - 2ab\cos C$

Law of Cosines

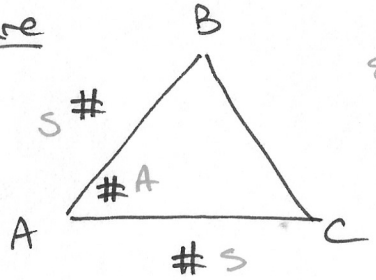
SAS

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$

Picture



SAS

angle between sides

$c = 10 \quad b = 7 \quad A = 47^\circ$

$C = \quad B = \quad a = ?\ 7.3$

$a^2 = 7^2 + 10^2 - 2(7)(10)\cos 47^\circ$

$\sqrt{a^2} = \sqrt{53.5202}$

$a = 7.3$

TO FIND AN ANGLE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \frac{-2bc\cos A}{-2bc}$$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

Ex: $A = 47^\circ$ $B = 44^\circ$ $C = 89^\circ$

$a = 7.3$ $b = 7$ $c = 10$

$$\cos C = \frac{10^2 - 7.3^2 - 7^2}{-2(7.3)(7)} \quad \cos^{-1}(\text{decimal})$$

Example: $A = 55^\circ$ $B = 10^\circ$ $C = 115^\circ$ ∴

$a = 9$ $b = 2$ $c = 10$

$$\cos B = \frac{2^2 - 9^2 - 10^2}{-2(9)(10)} \qquad B = 10^\circ$$

$$\cos A = \frac{9^2 - 2^2 - 10^2}{-2(2)(10)} \qquad A = 55^\circ$$

FINDING Area: 3 sides (Heron's)

$$K = \sqrt{s(s-b)(s-a)(s-c)}$$

where

$$s = \frac{1}{2}(a+b+c)$$

Ex: $a = 9$ $b = 2$ $c = 10$

$$s = \frac{1}{2}(9+2+10)$$

$$s = 10.5$$

$$K = \sqrt{10.5(10.5-2)(10.5-9)(10.5-10)}$$

$$10.5(8.5)(1.5)(.5)$$

$$K = 8.2\ u^2$$