

Pythagorean Identities

$$\textcircled{1} \quad \cos^2 \theta + \sin^2 \theta = 1$$

$$\textcircled{2} \quad 1 + \tan^2 \theta = \sec^2 \theta$$

$$\textcircled{3} \quad 1 + \cot^2 \theta = \csc^2 \theta$$

Example: given $\cos = \frac{4}{5}$; $\boxed{270^\circ < \theta < 360^\circ}$

find $\sin \theta =$ IV

$$\left(\frac{4}{5}\right)^2 + \sin^2 = 1$$

$$\frac{16}{25} + \sin^2 = 1 - \frac{16}{25}$$

$$-\frac{16}{25}$$

$$\sqrt{\sin^2} = \sqrt{\frac{9}{25}}$$

$$\sin \theta = -\frac{3}{5}$$

tells
positive
or
negative

Example: given $\sec = \frac{6}{5}$, $\boxed{0^\circ < \theta < 90^\circ}$

find $\tan \theta =$ I

$$1 + \tan^2 = \sec^2$$

$$1 + \tan^2 = \left(\frac{6}{5}\right)^2$$

$$1 + \tan^2 = \frac{36}{25} - 1$$

$$-1$$

$$\sqrt{\tan^2} = \sqrt{\frac{11}{25}}$$

$$\tan = \frac{\sqrt{11}}{5}$$

example: given $\cot \theta = \frac{3}{5}$, $\boxed{\pi < \theta < \frac{3\pi}{2}}$

find $\sin \theta =$ III

$$1 + \cot^2 = \csc^2$$

$$1 + \left(\frac{3}{5}\right)^2 = \csc^2$$

$$1 + \frac{9}{25} = \csc^2$$

$$\sqrt{\frac{34}{25}} = \sqrt{\csc^2}$$

$$\frac{\sqrt{34}}{5} = \csc$$

$$\sin = \frac{\theta}{\sqrt{34}}$$

$$\sin \theta = \frac{-5\sqrt{34}}{34}$$

ex 3: given $\sin \theta = \frac{3}{4}$, $\frac{\frac{\pi}{2} < \theta < \pi$, $\begin{matrix} 90^\circ & & 180^\circ \end{matrix}$

find $\cot \theta =$ \swarrow $\text{csc} \theta = \frac{4}{3}$

$$\cot^2 + 1 = \csc^2$$

$$\cot^2 + 1 = \left(\frac{4}{3}\right)^2$$

$$\cot^2 + 1 = \frac{16}{9} - 1$$

$$\underline{-1}$$

$$\sqrt{\cot^2} = \sqrt{\frac{7}{9}} \quad \cot \theta = -\frac{\sqrt{7}}{3}$$

ex 4: given $\tan \theta = \frac{10}{11}$, $\frac{\pi < \theta < \frac{3\pi}{2}}$, III

find $\cos \theta =$

$$\tan^2 + 1 = \sec^2$$

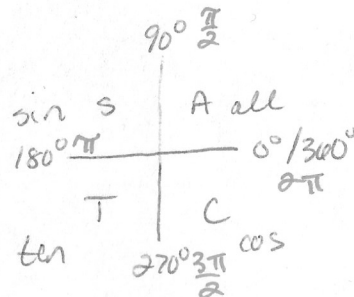
$$\left(\frac{10}{11}\right)^2 + 1 = \sec^2$$

$$\frac{100}{121} + 1 = \sec^2$$

$$\frac{221}{121} = \sec^2 \quad \sqrt{\cos^2} = \sqrt{\frac{121}{221}}$$

$$\cos = \frac{11}{\sqrt{221}}$$

$$\cos \theta = -\frac{11\sqrt{221}}{221}$$



ex: given $\cos \theta = -\frac{1}{10}$, $\frac{\frac{\pi}{2} < \theta < \pi$, II

find $\tan \theta =$ \swarrow $\text{sec} \theta = -10$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \tan^2 = (-10)^2$$

$$\sqrt{\tan^2} = \sqrt{99}$$

$$\tan = \sqrt{99} \quad \tan = -3\sqrt{11}$$

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