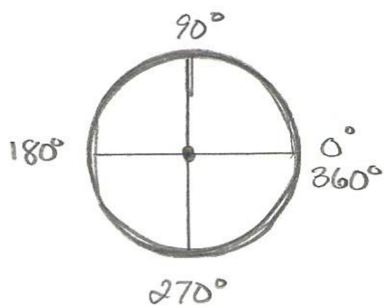
# Angles & Radians

**Radian:** a pure measure based on the radius of the ⊙.

- one radian is the angle made at the center of a ⊙ by an arc whose length is equal to the radius of the ⊙

- you can derive the radian measures for the special angles of the unit ⊙.

* think about the unit ⊙




- we know that one revolution is 360°
- from Geometry we know that the distance around the circle is the circumference or
  $C = 2\pi r$

$\therefore$ if radius is 1, we can conclude that $360° = 2\pi$

- so $360° = 2\pi$ radians

  or

  $180° = \pi$ radians

- thus, we can conclude that

1 degree $= \frac{\pi}{180}$ radians & 1 radian $= \frac{180}{\pi}$ degrees

or .017 or 57.3 degrees

Thus, we can convert between degrees & radians

- to convert to radians, take angle measure and multiply by $\frac{\pi}{180}$

ex: $40°$ $\quad 40 \times \frac{\pi}{180} = \frac{40\pi}{180} = \frac{2\pi}{9}$ radians

- to convert to degrees, take radian measure and multiply by $\frac{180}{\pi}$

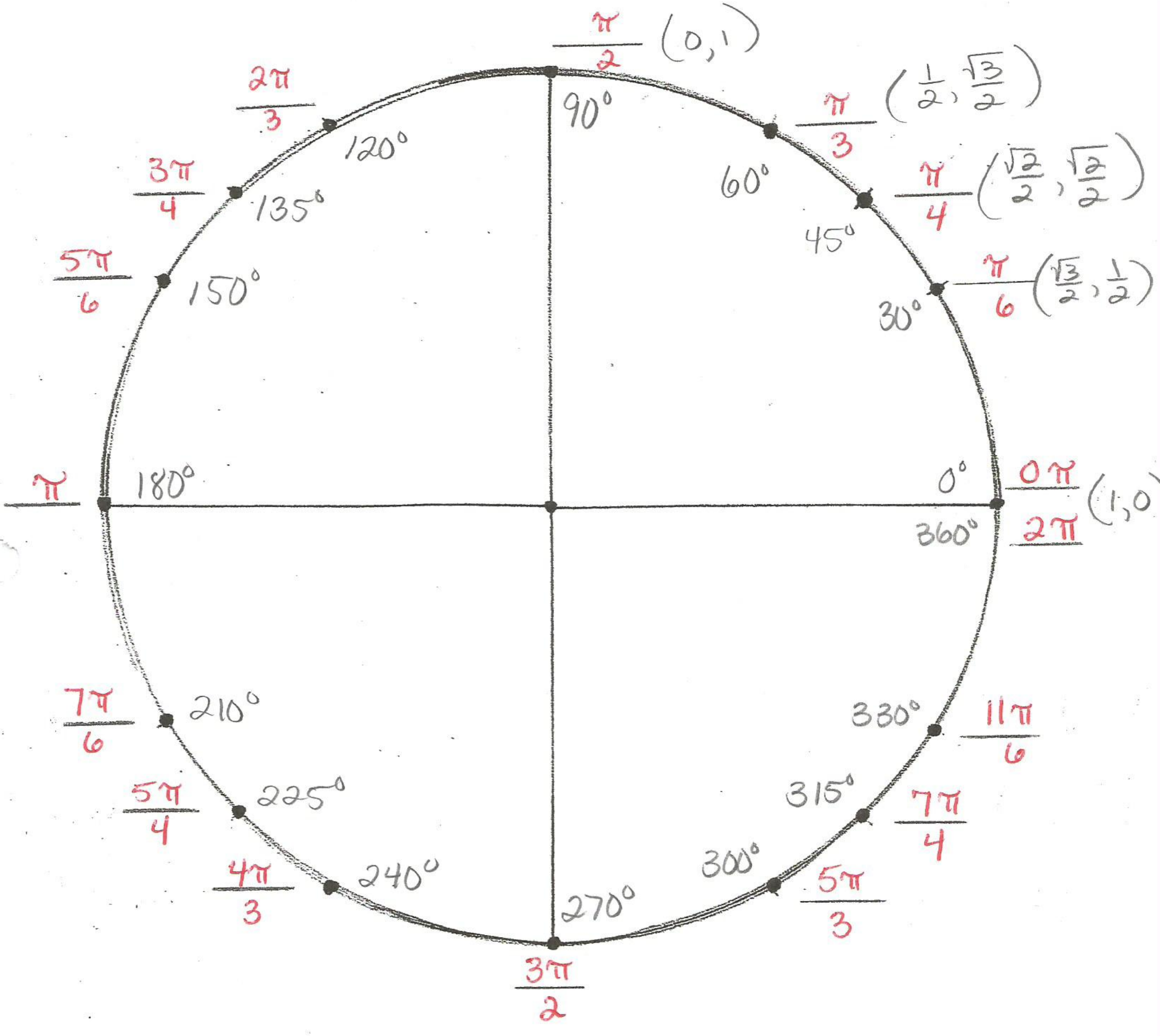
ex: $\frac{5\pi}{4}$ $\quad \frac{5\pi}{4} \times \frac{180}{\pi} = \frac{900\cancel{\pi}}{4\cancel{\pi}} = 225°$

ex: 1.5 radians $\quad 1.5 \times \frac{180}{\pi} = \frac{270}{\pi} = 85.9°$

- Now, we can convert the unit circle from degrees to radians by multiplying each angle by $\frac{\pi}{180}$

Note: radians of the unit circle have the same coordinates as the degrees

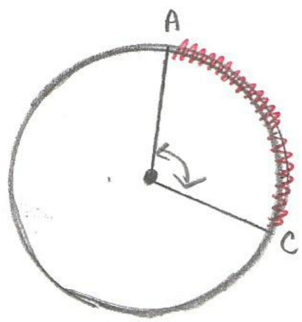
# The Unit Circle



# Radians & Angles

- knowing radians, we can use them for more precise measurements.

Given: what if we wanted to find the length of arc $\overset{\frown}{AC}$?




From Geometry: used a proportion to solve.

$$\frac{\overset{\frown}{AC}}{2\pi r} = \frac{m\angle}{360}$$

Now: Cross multiply $2\pi r$ we have

- $\overset{\frown}{AC} = 2\pi r \cdot \frac{m\angle}{360}$

- $\overset{\frown}{AC} = \pi r \cdot \frac{m\angle}{180}$ * 2 goes into 360 to get 180

- $\overset{\frown}{AC} = r \cdot \boxed{m\angle \cdot \frac{\pi}{180}}$ * rearranged

↳ this is converting to radian measure

∴ thus we have "Arc Length" formula

$$\boxed{s = r \cdot \theta}$$

arc length

radius of ⊙

radian measure of central ∠

# Arc length

Example: given $r = 4$ $\theta = \frac{3\pi}{2}$

$s = r \cdot \theta$

$s = 4 \cdot \frac{3\pi}{2} \Rightarrow \frac{12\pi}{2} \Rightarrow \boxed{6\pi \text{ or } 18.8}$

Example: given $r = 8$ $\theta = 200°$

1st: convert 200° to radians: $200° \cdot \frac{\pi}{180} \Rightarrow \frac{200\pi}{180} = \frac{10\pi}{9}$

$s = 8 \cdot \frac{10\pi}{9} \Rightarrow \boxed{\frac{80\pi}{9} \text{ or } 27.9}$

Example: given $s = 10\pi$ & $\theta = \frac{2\pi}{3}$ find "r"

$s = r \cdot \theta$

$10\pi = \frac{2\pi}{3} \cdot r$

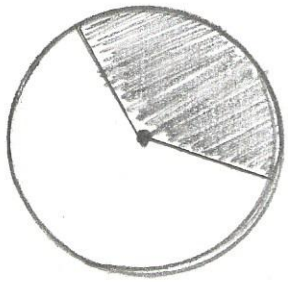
$\frac{3}{2\pi} \cdot 10\pi = \cancel{\frac{2\pi}{3}} \cdot r \cancel{\frac{3}{2\pi}}$

$\frac{30\pi}{2\pi} = r$

$15 = r$

# Area of Sector

- we can also use radians to find area



Sector: a portion of the circle

from Geometry: the proportion

area of sector → $\dfrac{A}{\pi r^2} = \dfrac{\overset{\frown}{Arc}}{2\pi r}$ ← Arc length

* remember we know arc length as $r \cdot \theta$

Now: $\dfrac{A}{\pi r^2} = \dfrac{r \cdot \theta}{2\pi r}$ cross multiply

$A = \pi r^2 \cdot \dfrac{r\theta}{2\pi r}$ * from here $\pi$ cancels & $r$ cancels

$A = \cancel{\pi} r^2 \cdot \dfrac{\cancel{r}\theta}{2\cancel{\pi}\cancel{r}}$

$\therefore$ $\boxed{A = \frac{1}{2} r^2 \theta}$ Area of Sector

area of sector

radius of $\theta$

radian measure of central $\angle$.

# Area of Sector

**Example:** Given $r = 14$ $\theta = \frac{\pi}{6}$

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} (14)^2 \cdot \frac{\pi}{6}$$

$$A = \frac{1}{2} (196) \frac{\pi}{6} \Rightarrow \frac{98\pi}{6} \Rightarrow \boxed{\frac{49\pi}{3} \text{ or } 51.3}$$

**Example:** Given $r = 4$ $\theta = \frac{7\pi}{4}$

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} \cdot 4^2 \cdot \frac{7\pi}{4}$$

$$A = \frac{1}{2} (16)\left(\frac{7\pi}{4}\right) \Rightarrow \frac{56\pi}{4} \Rightarrow \boxed{14\pi \text{ or } 44}$$