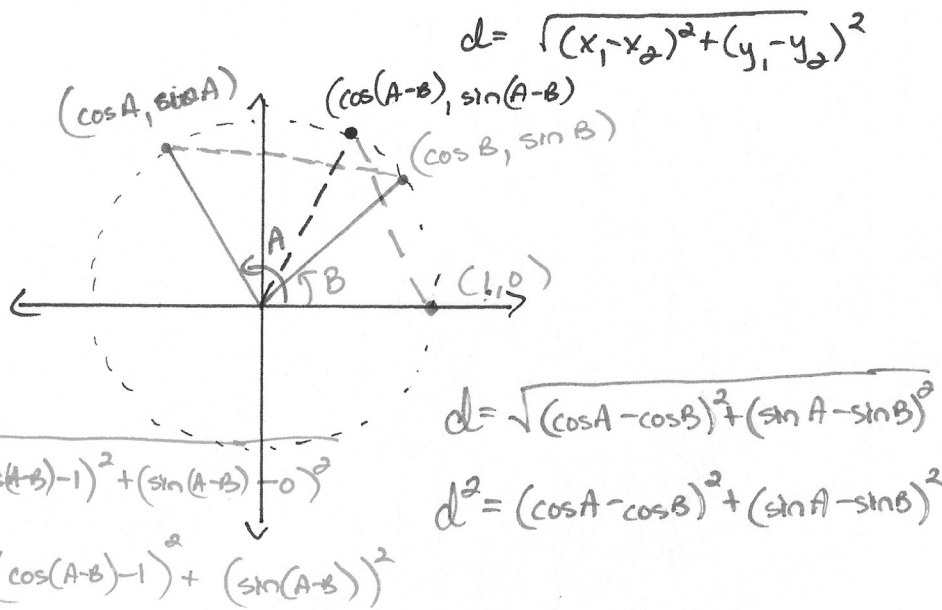


# Sum & Difference Identities



$$(\cos(A-B) - 1)^2 + (\sin(A-B))^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$$

$$\underbrace{\cos^2(A-B)} - 2\cos(A-B) + 1 + \underbrace{\sin^2(A-B)} = \underbrace{\cos^2 A - 2\cos A \cos B + \cos^2 B} + \underbrace{\sin^2 A - 2\sin A \sin B + \sin^2 B}$$

$$1 + (-2\cos(A-B)) + \sin^2(A-B) = 1 + (-2\cos A \cos B - 2\sin A \sin B) + \sin^2 A + \sin^2 B$$

$$\frac{-2\cos(A-B)}{-2} = \frac{-2\cos A \cos B - 2\sin A \sin B}{-2}$$

$$\boxed{\cos(A-B) = \cos A \cos B + \sin A \sin B}$$

$$\cos(A+B) = \cos(A-B) = \cos A \cos(-B) + \sin A \sin(-B)$$

$$\boxed{\cos(A+B) = \cos A \cos B - \sin A \sin B}$$

Think  $\sin \theta = \cos(90 - \theta)$

$$\sin(A+B) = \cos(90 - (A+B))$$

$$\cos((90-A) - B)$$

$$= \cos(90-A) \cos B + \sin(90-A) \sin B$$

$$\boxed{\sin(A+B) = \sin A \cos B + \cos A \sin B}$$

$$\sin(A-B) = \sin A \cos(-B) + \cos A (\sin(-B))$$

$$\boxed{\sin(A-B) = \sin A \cos B - \cos A \sin B}$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$\boxed{-\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

Algebra III  
Sum & Difference Identities

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Find each exact value if  $0 < x < \frac{\pi}{2}$  and  $0 < y < \frac{\pi}{2}$

1)  $\sin(x + y)$  if  $\cos x = \frac{8}{17}$  and  $\sin y = \frac{12}{37}$

2)  $\cos(x - y)$  if  $\cos x = \frac{3}{5}$  and  $\cos y = \frac{4}{5}$

3)  $\tan(x - y)$  if  $\sin x = \frac{8}{17}$  and  $\cos y = \frac{3}{5}$

4)  $\cos(x + y)$  if  $\tan x = \frac{5}{3}$  and  $\sin y = \frac{1}{3}$

①  $\sin(x+y) = \sin x \cos y + \cos x \sin y$   
 $\left(\frac{15}{17}\right)\left(\frac{35}{37}\right) + \left(\frac{8}{17}\right)\left(\frac{12}{37}\right)$   
 $17^2 - 8^2 = 225$   $37^2 - 12^2 = 1025$   $a^2 + b^2 = c^2$   $H^2$   $a^2 + 12^2 = 37^2$

$$\frac{525}{629} + \frac{96}{629} = \boxed{\frac{621}{629}}$$

②  $\cos(x-y) = \cos x \cos y + \sin x \sin y$   
 $\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$   
 $5^2 - 3^2$

$$\frac{12}{25} + \frac{12}{25} = \boxed{\frac{24}{25}}$$

④  $\cos(x+y) = \cos x \cos y - \sin x \sin y$   
 $\left(\frac{3\sqrt{34}}{34}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{5\sqrt{34}}{34}\right)\left(\frac{1}{3}\right)$   
 $3^2 - 1^2 = 8$

$$\frac{4\sqrt{17}}{102} - \frac{5\sqrt{34}}{102}$$

$$\boxed{\frac{12\sqrt{17} - 5\sqrt{34}}{102}}$$

$$\textcircled{3} \quad \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\frac{\frac{8}{15} - \frac{4}{3}}{1 + \left(\frac{8}{15}\right)\left(\frac{4}{3}\right)}$$

$$= \frac{-12}{15}$$

$$1 + \frac{32}{45}$$

$$= \frac{-12}{15} \cdot \frac{77}{45}$$

$$\frac{-12}{15} \cdot \frac{77}{45}$$

$$= \frac{-36}{77}$$

$$\sin x = \frac{8}{17} \quad 17^2 - 8^2$$

$$\tan x = \frac{8}{15}$$

$$\cos y = \frac{3}{5} \quad 5^2 - 3^2$$

$$\tan y = \frac{4}{3}$$